

PHYS-100C, Homework #4 Solutions

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2:30 PM

10.1 $L = \nabla \cdot A + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

$$\nabla^2 V + \frac{\partial L}{\partial t} = \nabla^2 V - \cancel{\mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2}} + \frac{\partial}{\partial t} (\nabla \cdot A) +$$

$$\cancel{+ \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2}} = \nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot A) = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 A - \nabla L = \nabla^2 A - \mu_0 \epsilon_0 \frac{\partial^2 A}{\partial t^2} - \nabla \left(\nabla \cdot A + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} \right) =$$

$$= -\mu_0 \vec{J}$$

10.3 $V = 0$
 $A = -\frac{1}{4\pi\epsilon_0} \frac{q+t}{r^2} \hat{r}$

$$E = -\cancel{\nabla V} - \frac{\partial A}{\partial t} = \frac{q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2}$$

Field from ^{stationary} point charge, at $r=0$

$$\rho = q \cdot \delta^3(r), \quad \vec{J} = 0$$

$$\vec{B} = \nabla \times A = 0$$

10.5 $V' = V - \frac{\partial \lambda}{\partial t} = \frac{q}{4\pi\epsilon_0 r}$

$$A' = A + \nabla \lambda = -\frac{q+t}{4\pi\epsilon_0 r^2} \hat{r} + \left(-\frac{q+t}{4\pi\epsilon_0} \right) \left(-\frac{\hat{r}}{r^2} \right) =$$

$$= 0$$

10.7 $\phi = \nabla \cdot \mathbf{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

How does ϕ change under gauge transformation?

$$\begin{aligned} \phi' &= \nabla \cdot \mathbf{A}' + \frac{1}{c^2} \cdot \frac{\partial V'}{\partial t} = \nabla \cdot \mathbf{A} + \nabla^2 \lambda + \frac{1}{c^2} \frac{\partial V}{\partial t} - \\ &\quad - \frac{1}{c^2} \frac{\partial^2 \lambda}{\partial t^2} = \phi + \underbrace{\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right)}_{\text{"Square Squared"} \rightarrow \square^2} \lambda \end{aligned}$$

"Square Squared" $\rightarrow \square^2$
as some of you call it

For any given (scalar) function ρ (charge distribution) we know exists a potential field V , such that:

$$\square^2 V = -\frac{\rho}{\epsilon_0}$$

If we replaced $\frac{\rho}{\epsilon_0}$ by ϕ , another random scalar field, there exists λ^* such that

$$\square^2 \lambda^* = -\phi$$

Using that λ^* for gauge tr.:

$$\phi' = \phi + \square^2 \lambda^* = \phi - \phi = 0$$

for all (\vec{r}, t) .

We can always make $V \leq 0$

$$\frac{\partial \lambda}{\partial t} = V \Rightarrow \lambda = \int V \cdot dt' + \text{const}$$

Cannot generally make $A=0 \Rightarrow$
 $B = \nabla \times A = 0$